Dark energy from non-unitarity in quantum theory

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We consider a scheme whereby it is possible to reconcile semi-classical Einstein's equation with the violation of the conservation of the expectation value of energy-momentum that is associated with dynamical reduction theories of the quantum state for matter. The very interesting out-shot of the formulation is the appearance of a nontrivial contribution to an effective cosmological constant (which is not strictly constant). This opens the possibility of using models for dynamical collapse of the wave function to compute its value. Another interesting implication of our analysis is that tiny violations of energy-momentum conservation with negligible local effects can become very important on cosmological scales at late times.

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I. INTRODUCTION

Since the discovery of the acceleration in the universe's expansion [1, 2], almost two decades ago, there has been a puzzlement about the strange value of the corresponding cosmological constant Λ; the simplest, and so far most successful, theoretical model that could account for the observed behaviour. The origin of this puzzle is that, within the usual framework, the only seemingly natural values that Λ could take are either zero, or a value which is 120 orders of magnitude larger than the one indicated by observations $\Lambda^{\rm obs} \approx 1.1\ 10^{-52}\ {\rm m}^{-2}$ [3][1].

Here we present a scenario where something very similar to a cosmological constant emerges from considering violation of energy conservation associated with modifications of quantum mechanics involving non-unitary evolution. Such modifications have been suggested in the context of information loss during black holes evaporation, and in theories with spontaneous dynamical reduction of the quantum state of matter [7], which have been developed to address the so-called measurement problem of quantum mechanics [8].

One of the most serious difficulties faced by such proposals relates to their consistency (or lack thereof) with the gravitational interaction[2]. Another issue that has lead to concerns is whether such proposals entail unacceptably large violations of energy conservation or violations of causality. A first analysis carried out in [10] argued that these problems were in fact unavoidable while a latter study of this issue showed that they can be bypassed by suitable choices for the theory [11].

The present work gets inspiration from those studies, and focuses on the resolution of the apparent tension between dynamical reduction theories and metric theories of gravitation. Even if the application of our framework in this paper is based on energy-conservation-violations stemming from modifications of quantum mechanics, on a more general ground, our work proposes a new paradigm for analyzing the dark energy puzzle in cosmology, that identifies potential violations of energy-momentum conservation (that could be postulated on a simply phenomenological ground) as a source of dark energy.

The paper is organized as follows: In the following section we discuss a natural framework where (a certain type of) violation of energy-momentum conservation is consistent with a metric theory of gravity, closely related with general relativity. In Section III we describe the general features of modifications of quantum mechanics naturally leading to violations of energy-momentum conservation. In Section IV we study a concrete model of modified quantum mechanics, and show that it leads to a contribution to dark energy that is steady at late times, and has a magnitude of the same order as the measured cosmological constant (with the opposite sign). We conclude with a discussion of our results in Section V.

II. VIOLATION OF ENERGY-MOMENTUM CONSERVATION IN UNIMODULAR GRAVITY

In general relativity, local energy-momentum conservation is a consequence of the field equations, both at classical and semi-classical levels. Let us review by starting from the semi-classical version of Einstein's equation

[1] There have been a number of proposals where this value is not in fact constant: there are schemes known as quint-essence where a scalar filed with special dynamics is able to track the density of ordinary matter [4], and there is in fact a scheme proposed by R. Sorkin [5] where a value of the right order of magnitude is inferred for the current value, but which cannot really incorporate, in a self-consistent manner, its required evolution with the universe's aging process. In [6], a scheme where, by feat, the cosmological constant is assumed to be able to vary in response to matter's energy-momentum non-conservation has been suggested.

[2] Recently an interesting proposal was put forward that seems to resolves the issues in the Newtonian gravity setting [9].

$$R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G \langle T_{ab} \rangle \ , \tag{1}$$

where $\langle T_{ab} \rangle$ is the expectation value of the (renormalized) energy-momentum tensor operator in the corresponding quantum state of the matter fields. Due to Bianchi identities the geometric side (*l.h.s.*) is divergence free. So (1) is simply incompatible with models in which the dynamics of the state departs from the standard unitary evolution provided by Schrödinger's equation, as that leads in general to violation of the condition $\nabla^b \langle T_{ab} \rangle = 0$ [12][3].

The previous difficulty can be circumvented in an rather elegant way by considering a simple modification of general relativity, already evoked by Einstein in 1919 when trying to construct geometric account for elementary particles in terms of radiation fields [14]. He proposed the trace-free equation

$$R_{ab} - \frac{1}{4} R g_{ab} = \frac{8\pi G}{c^4} \left(T_{ab} - \frac{1}{4} T g_{ab} \right), \tag{2}$$

which has been rediscovered several times, and is called today unimodular gravity (see [15] and references therein). Unimodular gravity can be derived from the Einstein-Hilbert action by restricting to variations that preserve the (four) volume-form, namely those for which $g_{ab}\delta g^{ab} = 0$. For an infinitesimal diffeomorphism represented by the vector field ξ^a this condition becomes

$$\nabla_a \xi^a = 0 \ . \tag{3}$$

If the matter action is invariant only under diffeomorphisms satisfying (3), then one has

$$\delta S_m = \int_M \sqrt{-g} T_{ab} \nabla^a \xi^b dx^4 = \int_M \sqrt{-g} \phi \nabla_a \xi^a dx^4, \tag{4}$$

for some scalar field ϕ and where we used that $\delta S_m / \delta g^{ab} = -8\pi G \sqrt{-g} T_{ab}$. Assuming ξ^a has compact support, and integrating by parts we get

$$J_b \equiv \nabla^a T_{ab} = \nabla_b \phi, \tag{5}$$

where we have introduced the energy-momentum violation current J_b. The previous equation tells us that violations of energy-momentum conservation in unimodular gravity must be of the *integrable* type $dJ = 0$[4].

Hence, unimodular gravity is *a priori* only invariant under diffeomorphisms preserving the space-time volume. However, it becomes indistinguishable from classical general relativity (including general covariance), as soon as conservation of the energy-momentum tensor ($J = 0$) is added as an additional requirement on the matter action.

On the contrary, if the matter action is only invariant under volume preserving diffeomorphisms then $J \neq 0$ will introduce deviations from general relativity. As we argue below such deviations can arise naturally at the quantum level; however, on a phenomenological ground, they could also be introduced classically in the choice of the matter action S_m.

An important feature of unimodular gravity appears in the semi-classical regime: the vacuum fluctuations of the energy-momentum tensor do not gravitate, thus removing the need to contemplate the enormous discrepancy between the observed values of the effective cosmological constant and the standard estimates for the vacuum energy [15, 17, 18]. Another feature of unimodular gravity is that the quantum theory, defined from the canonical perspective, admits a more familiar description in terms that considerably dilute the weight of the *problem of time* in quantum gravity [19, 20].

Let us now consider the semi-classical version of equation (2), where the energy-momentum tensor and its trace are replaced by the corresponding expectation values in the quantum state of the matter fields. Using Bianchi identities, one then deduces

$$\frac{1}{4} \nabla_a R = \frac{8\pi G}{c^4} \left(\nabla^b \langle T_{ab} \rangle - \frac{1}{4} \nabla_a \langle T \rangle \right). \tag{6}$$

In terms of the mean current $J_a = \nabla^b \langle T_{ab} \rangle$ we get [5]

$$\frac{1}{4} \left(R + \frac{8\pi G}{c^4} \langle T \rangle \right) = \Lambda_{-\infty} + \frac{8\pi G}{c^4} \int_\ell J \ ,$$

where $\Lambda_{-\infty}$ is a constant of integration, and ℓ is a path in space-time connecting an initial reference-event to the point where the *l.h.s.* is evaluated. Then the semi-classical version of (2) can be recast as

$$R_{ab} - \frac{1}{2} R g_{ab} + \left(\Lambda_{-\infty} + \frac{8\pi G}{c^4} \int_\ell J \right) g_{ab} = \frac{8\pi G}{c^4} \langle T_{ab} \rangle. \tag{7}$$

As said previously, if one assumes conservation of the expectation value of the stress-energy tensor, namely $J = 0$, then (7) reduces to Einstein's equation, where the cosmological constant $\Lambda_{-\infty}$ is now interpreted as an integration constant. However, in contrast to general relativity, energy-momentum conservation is not imposed *a priori* in connection with the equation of motion (2), but is an extra assumption that can be relaxed.

In the cosmological setting, the effective cosmological "constant"

$$\Lambda^{\text{eff}}(t) = \Lambda_{-\infty} + \frac{8\pi G}{c^4} \int^t J \ , \tag{8}$$

[3] A detailed formalism which circumvented this problem while viewing the above equation only as an "effective description" (analogous to the Navier Stokes equations describing a fluid), subject to instantaneous breakdowns was considered in [13].

[4] An intriguing possibility, for modified dynamics proposals where this condition does not hold, would be to introduce space-time torsion [16].

[5] Of course, the collapse dynamics or modified quantum evolution is restricted for consistency with (6) so that $dJ = 0$.

at any stage in the universe's evolution, is related to a possible violation of energy-momentum conservation in the past history of the universe. In the last section, we will show that small violations of energy-momentum conservation—that might remain inaccessible to current tests of local physics—can nevertheless have important cosmological effects at late times, in the form of the non-trivial contribution to the present value of the cosmological constant (8).

As such violation of energy-momentum conservation is a central feature of modified (non-unitary) theories of quantum mechanics with spontaneous collapse, the present framework is the natural one to describe these models in the gravitational context. On a more general ground, and beyond the application of the present framework to those particular models of modified quantum mechanics, our work proposes a new paradigm for analyzing the dark energy puzzle in cosmology, that identifies potential violations of energy-momentum conservation with a source of dark energy.

III. NON-UNITARY QUANTUM DYNAMICS

There are various reasons to consider possible deviations from the standard quantum dynamics. An important piece of the motivation is the *measurement problem*. This issue goes back to 1935, when Schrödinger published his famous "cat thought-experiment" [21], illustrating the tension between superposition of states—coherent or not—as predicted by quantum mechanics, and the macroscopic world we all live in; nowadays, often referred as the *measurement problem* [8]. Another reason (linking these potential modifications directly to gravity) emerges from questions related to the fate of information during the Hawking evaporation of a black hole. When studying possible modifications of quantum theory it seems natural that, in order to recover Born's rules for probabilities of experimental outcomes, and thus have an empirically viable candidate theory, non-linearity and stochasticity should be key features of the hypothetical fundamental dynamics.

Taking these motivations into account, there is a wide class of phenomenological models—those with a Markovian evolution—that can all be described[6] by the same type of evolution equation for the density matrix $\hat{\rho}$: the so-called Kossakowski-Lindblad equation [22, 23]

$$\dot{\hat{\rho}} = -i[\hat{H}, \hat{\rho}] - \frac{1}{2}\sum_{\alpha} \lambda_\alpha [\hat{Q}_\alpha, [\hat{Q}_\alpha, \hat{\rho}]] \ , \tag{9}$$

where $\hat{H}$ is the standard Schrödinger Hamiltonian operator , $\{\hat{Q}_\alpha\}$ are the (hermitian) operators characterizing the modified dynamics, and $\{\lambda_\alpha\}$ are suitable parameters determining the strength of the new effects. Such equation has been used to describe a possible non-unitary evolution [10, 11] induced by the creation and evaporation of black holes[7] in the context of Hawking's information puzzle [24], while it also appears in the description of modifications of quantum mechanics with spontaneous stochastic collapse [25, 26]. It has been argued by Penrose [27] that the two apparently different contexts could actually be related in a more fundamental description of quantum gravitational phenomena. In all these cases, a central feature of this equation is that the average energy $E \equiv \mathrm{Tr}[\hat{\rho}\hat{H}]$ is generically not constant.

Recent studies naturally led us to consider equation (9) and the possibility of violations of energy-momentum conservation. For instance a series of papers involving one of us considers the proposal that the information loss during the evaporation of black holes and the deviation from unitarity associated with resolutions of the measurement problem would be just two faces of the same thing, leading to what seems to be an overall coherent picture [28]. Further works involving two of us explore the related questions in the context of the origin of cosmic structures [29]. On a more conservative ground, in the context of a theory of quantum gravity that is unitary (in the sense of being constructed by the application of a unitary quantization framework), but where spacetime geometry is emergent from pre-geometric quantum structures at the Planck scale, one expects that the effective description of low energy degrees of freedom, as evolving in a smooth spacetime geometry, would be described by an equation of the type (9). A scenario based on this idea has been proposed as a possible framework for understanding the information puzzle in black hole evaporation [30]. It is possible that violations of energy-momentum conservation of the type discussed here could arise in such context as well. Indeed modifications of similar kind are shown to arise from fundamental discreteness in models considered in [31, 32].

IV. A SIMPLE EXAMPLE: CONTINUOUS SPONTANEOUS LOCALIZATION OF BARYONIC MATTER

The point of view we take in this section, once again, assumes that the collapse of the wave-function is a real

[6] The fundamental evolution equation is often presented as a stochastic differential equation controlling the evolution of individual physical systems, and containing explicit stochastic terms, however it is often convenient to consider the evolution of large ensembles of such systems. At that point the evolution dynamics takes the form presented here.

[7] In [10] it is argued that (9) should be dismissed as it would lead to unacceptably large violations of energy conservation or to dramatic violations of causality. However, further analysis [11] showed that those arguments were not robust and that (9) is actually compatible with violations of energy-momentum conservation that are sufficiently small not to run into conflict with observations.

physical process, and that Schrödinger's equation is only an approximation of a more fundamental dynamics from which classical objective reality emerges in the suitable limit.

However, due to the lack of precise knowledge of such fundamental description we adopt a phenomenological view, and concentrate on models of dynamical collapse.

A. Spontaneous localization models

The history of models of this sort goes back to Pearle [25] in 1976. The first truly viable one was produced shortly after, by Ghirardi, Rimini, and Weber (GRW) in [26], which characterized the dynamics in terms of spontaneous discrete events of localization of a particle's wave function, with a position uncertainty r_c occurring, at a time rate given by a second fundamental parameter λ. Further developments resulted in the continuous spontaneous localization (CSL) model [33, 34] which replaced the discrete collapse dynamics by continuous albeit stochastic dynamical law. This theory can be characterized by the same parameters r_c and λ as the GRW theory, with the only difference that the time rate λ is transformed into a coupling constant controlling the magnitude of the non-unitary stochastic terms in the dynamics.

The mathematical description of the modified quantum dynamical evolution, for a single quantum degree of freedom, as dictated by CSL, is specified by two equations: a modified Schrödinger equation, whose solution is

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{Q}]^2 \right]} |\psi, 0\rangle, \qquad (10)$$

where $\hat{\mathcal{T}}$ is the time-ordering operator, $w(t)$ is a random, white noise type classical function of time whose probability is given by the second equation

$$\mathrm{PD}w(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda_0/dt}}, \qquad (11)$$

defining the probability of a realization of the stochastic function $w(t)$. In the continuum limit, averaged density matrix defined as

$$\hat{\rho} \equiv \int \mathrm{PD}w(t)\ |\psi, t\rangle_{ww}\langle\psi, t|, \qquad (12)$$

satisfies an evolution equation of type (9).

Thus the standard Schrödinger evolution and the corresponding measurement of the observable $\hat{Q}$ are unified. For non-relativistic quantum mechanics of a single particle, the proposal assumes that, (without invoking any measurement) $\vec{Q}$ is a suitably smeared version of the position operator over a region of radius r_c. When this is generalized to multi-particle systems, and everything including apparatuses are treated quantum mechanically, the theory is thought to successfully address the measurement problem [8]. The basic workings of the theory ensure that for a single particle in isolation, or even in the case of few interacting particles, the modified quantum evolution theory is essentially indistinguishable from that provided by Schrödinger's equation (for time durations shorter that λ^{-1}). However, when the number of particles involved starts approaching Avogadro's number, a situation that would include among others, any of the settings we normally describe as a measurement, and which involves a macroscopic device with a pointer made up of a large enough number of particles interacting with a simple, few particle, subsystem, which one would normally refer to, as the "quantum system", the modified dynamics would become dominant driving the complete system to one of the states corresponding to a relatively sharp localization of the pointer's center of mass. This in essence ensures that small systems are well described by standard quantum mechanics, while large enough systems will never be found in Schrödinger's cat type of states, and that moreover , systems involving interaction of both kinds of sub-systems behave in a way that corresponds to the Born rule for measurement processes in the Copenhagen version of quantum mechanics. For all this to work appropriately at the quantitative level, the collapse parameter λ must be small enough not to conflict with QM in its tested domain of applicability, and big enough to result in rapid localization of "macroscopic objects".

A review on recent progress in collapse models and on experimental constraints can be found here [35, 36]. Recently, there have been important advances in making these types of theory compatible with special relativity with three distinct models put forward in [37].

In all these models, the average energy, i.e., the expectation value of the standard quantum Hamiltonian, is not preserved under time evolution. In what follows, we focus only on the mass-proportional CSL model (i.e., the $\hat{Q}$ operator in (10) is a (smeared) mass-density operator). In that case, one predicts energy creation proportional to the mass of the system [38]:

$$\frac{dE}{dt} = \xi M c^2 \qquad \text{with} \qquad \xi = \frac{3}{16\pi^2}\lambda \left(\frac{r_N}{r_c}\right)^2 , \qquad (13)$$

where $r_N = h/(m_N c) \approx 1.32\ 10^{-15}$m is the neutron Compton wave-length. The current experimental constraints for this specific collapse model give (cf. [39] and figure 1)

$$3.3\ 10^{-42}\ \mathrm{s}^{-1} \leq \xi \leq 2.8\ 10^{-29}\ \mathrm{s}^{-1}\ . \qquad (14)$$

B. Implementation in cosmology

Equation (13) together with (7) pave the road for the study of the effects in cosmology of energy conservation violations, associated with dynamical reduction theories.

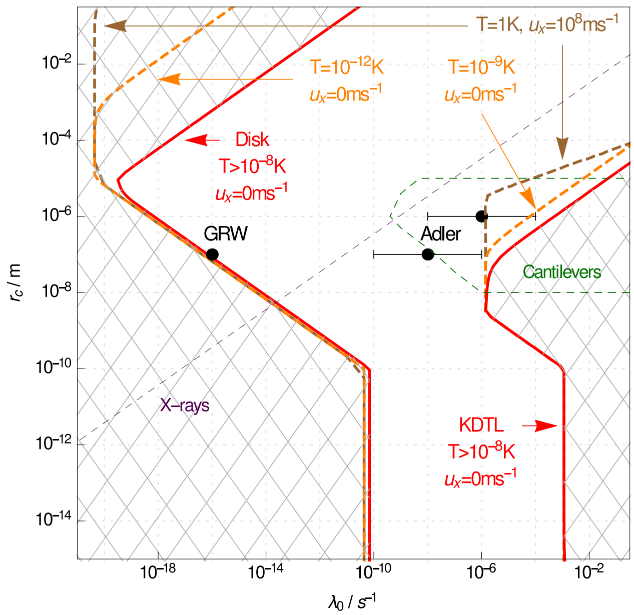

Figure 1. Experimental data. The figure reproduced here is courtesy of the authors of [40] which should be consulted for all the details.

Let's consider an homogeneous, isotropic and spatially flat Friedman-Lemaître-Robertson-Walker universe

$$ds^2 = -c^2 dt^2 + a^2(t) d\vec{x}^2,$$

In this setting we want to compute the time component of J_a due to spontaneous localization of baryonic matter (no collapse model is available for radiation and dark matter), which for the relevant situation we identified as a pressure-less perfect fluid. Using (13) , $J_t = -\xi \rho^{\mathrm{b}}$, where $\rho^{\mathrm{b}} = \rho_0^{\mathrm{b}} \left(a_0/a\right)^3$ is the baryon mass density.

Introducing Hubble rate $H = \dot{a}/a$ in (8), one can write

$$\Lambda^{\mathrm{eff}}(a_f) = \Lambda_{-\infty} - 3\Omega_0^{\mathrm{b}} \frac{H_0 \xi}{c^2} \left[\int_{a_i}^{a_f} \frac{H_0}{H(a)} \frac{a_0^3}{a^4}\, da \right], \quad (15)$$

where the index 0 denotes for the values of cosmological parameters today.

The contribution to the integral in (15) is strongly dominated by early times (small values of the scale factor a). As we consider the spontaneous localization of baryons, the natural initial time to choose is the hadronization epoch[8], at $a_0/a_h \approx T_{\mathrm{QCD}}/T_{\mathrm{CMB}} \approx 7\,10^{11}$ which is well inside the radiation-dominated era. An estimate of the effective cosmological constant at late time is then provided by

$$\Lambda^{\mathrm{eff}}(a_f \gg a_h) \approx \Lambda_{-\infty} - 3 \frac{\Omega_0^{\mathrm{b}}}{\sqrt{\Omega_0^{\mathrm{r}}}} \frac{H_0 \xi}{c^2} \frac{a_0}{a_h} \quad (16)$$

[8] We chose the mass-proportional CSL model and most of the mass of baryons comes from strong interaction.

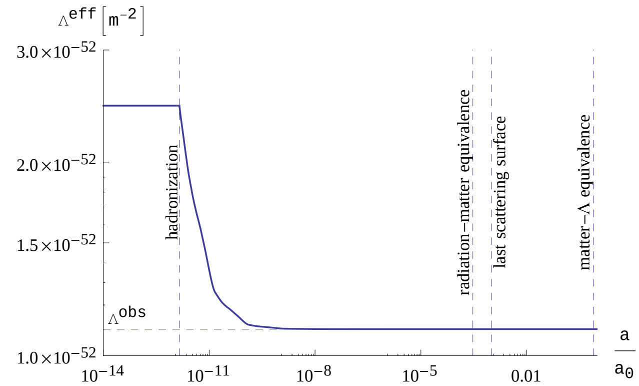

Figure 2. Effective cosmological constant induced by wave-function collapse of baryons, using mass-proportional CSL model with $\xi = 10^{-33}$ s^{-1}.

Using standard values for the cosmological parameters [41] Ω_0^{b}, Ω_0^{r} and H_0, we find that spontaneous localization induces a variation of the effective cosmological constant

$$\Delta\Lambda_0^{\mathrm{eff}} = \Lambda^{\mathrm{eff}}(a_0) - \Lambda_{-\infty} \approx -\left(\frac{\xi}{4\;10^{-31}\;\mathrm{s}^{-1}}\right)\Lambda^{\mathrm{obs}} \quad (17)$$

The contribution to the effective cosmological constant from baryonic collapse can be of the same order as the current value of the dark energy $\Lambda^{\mathrm{obs}} \approx 1.1\;10^{-52}\mathrm{m}^{-2}$, for allowed values of parameters r_C and λ, but it has the opposite sign. The numerical resolution of the equation (15), with $(H(a)/H_0)^2 = \Omega_0^{\mathrm{r}}(a_0/a)^4 + \Omega_0^{\mathrm{m}}(a_0/a)^3 + \Lambda^{\mathrm{eff}}(a)c^2/3H_0^2$, confirms the estimate (16). In Figure 2, we plot the strength of the dark energy contribution coming from our scenario as a function of a/a_0. We see how quickly it stabilizes to a constant value.

On the cosmological side, some approximations have been done. Firstly, the contribution to Ω^r of particles like electrons, muons or pions, which were relativistic at the hadronization epoch has been neglected. This would affect (16) by a numerical factor of order 1. Secondly, the quark-gluon plasma to hadron gas transition has been assumed to be instantaneous and to occur at T_{QCD}. As the contribution to the effective cosmological constant from baryonic matter collapse is dominated by the hadronization time, a modification of the phase transition could have a large effect on the value of Λ^{eff}. Lastly, the back-reaction from energy creation on the stress-energy tensor T_{ab} has not been taken into account.

Let's assume that the kinetic energy (13), created by spontaneous localization of baryons, is mostly transferred to photons (because of equipartition and the large number of photons), then the continuity equation for photons has to be modified according to $\dot{\rho}^{\gamma} + 4H\rho^{\gamma} = \xi\rho^{b}$. Neglecting any other relativistic fluids during radiation domination we have $(\dot{a}/a)^2 \approx 8\pi G\rho_{\gamma}/3$, so the continuity equation for photons reads

$$\frac{d\rho^{\gamma}}{da} + 4\frac{\rho^{\gamma}}{a} - \xi\rho_0^b \frac{a_0^3}{a^4}\sqrt{\frac{3}{8\pi G\rho^{\gamma}}} = 0 \quad (18)$$

This equation can be solved exactly, and the solution is

$$\rho^{\gamma}(a) = \rho_h^{\gamma} \frac{a_h^4}{a^4} \left[1 + \alpha \left(a^3 - a_h^3\right)\right]^{\frac{2}{3}} , \tag{19}$$

where ρ_h^{γ} is the photon density at the hadronization time and

$$\alpha = \frac{1}{2} \xi \frac{\rho_0^b a_0^3}{\rho_h^{\gamma} a_h^4} \sqrt{\frac{3}{8\pi G \rho_h^{\gamma} a_h^4}} \tag{20}$$

The departure from usual equation of state can be estimated at the end of radiation era ($z_{\rm eq} \sim 3000$):

$$\alpha a_{\rm eq}^3 = \frac{\Omega_0^{\rm b}}{\sqrt{2}\Omega_0^{\rm m}} \frac{\xi}{H_{\rm eq}} = \frac{\Omega_0^{\rm b}}{2(z_{\rm eq}\Omega_0^{\rm m})^{3/2}} \frac{\xi}{H_0} < 10^{-17} . \tag{21}$$

For late times, a similar treatment leads to the conclusion that this energy production does not alter the dynamics which is entirely dominated by the cosmological constant.

This computation directly shows that the formula used for the Hubble rate is justified. More importantly, it indicates that a violation of energy conservation can induce large effects on cosmological scale (the cosmological constant is dominating today!) without leading to a significant departure from the equation of state for matter.

V. DISCUSSION

We have shown that, in unimodular gravity, the non-conservation of energy due to spontaneous localization induces an effective cosmological constant. We have seen that the contribution coming from ordinary matter since hadronization can be of the same order of magnitude as the value extracted from current observations (high red shift supernovas [1, 2] and the Cosmic Microwave background [42]), but it comes with the opposite sign. While it is clear therefore that this can not be the whole story, we find it encouraging that the estimates we are able to perform at this time, with some degree of reliability, may have physically relevant consequences. If the observed value of the dark energy is to be explained within this context, without invoking an *ad hoc* cosmological constant (i.e., when setting $\Lambda_{-\infty} = 0$), it is clear that the contribution from the relativistic particles, which in fact dominate over the baryons in their contribution to the overall energy density in both the post-hadronization pre-matter domination regime that we have considered, as well as the earlier cosmological eras, should be associated with a loss of energy, and thus a positive value of J_t. At this point, the relativistic versions of dynamical reduction theories which could be applied to the radiation and highly relativistic particles, or more precisely to corresponding quantum fields, are unfortunately not sufficiently developed to allow a reliable analysis of the issue.

However, we can point to some simple considerations indicating that feasibility of endothermic collapse processes, in contrast to the exothermic collapse process associated with spatial localization of non-relativistic particles that we have considered so far.

One possibility is provided by the effects of interactions. The general idea is the following: Free particles that undergo localization which might be envisioned a la GRW by multiplication of the wave function by a gaussian function of width r_c. This leads either to no net change in the particle energy, when the position uncertainty was already smaller than r_c before the collapse, or to an increase in the particle's kinetic energy associated with the enlargement of the momentum uncertainty of the new wave packet. On the other hand, when particles are subjected to relatively strong attractive interactions, the localization can lead to an increase of the negative energy associated with the effective potential, and thus to a net decrease in the particle's energy. One very interesting option of an effect of this sort is provided precisely by the phase transition taking the early quark gluon plasma dominating the universe at energies above 1 GeV, where the asymptotic freedom of the strong interaction is well established, to the hadronic phase, at temperatures below say 100 MeV, where confinement becomes dominant. The point is that as the temperature decreases it becomes energetically favorable to form color confined hadrons, and confinment is closely tied to localization of the parts. This hadronization process in cosmology has been the subject of various studies (see for instance [43]) which are unfortunately limited by the serious difficulties of QCD calculations in the strong interaction regime. Furthermore the incorporation of modified quantum theories in this regime has not even been attempted.

A second possibility is tied to the likelihood that the collapse dynamics, while favoring position localized states in the non-relativistic many particle quantum setting, would favor a very different kind of states in the highly relativistic regime, or in the regimes where a full quantum filed treatment is required. In this regard we point out for instance the analysis of the inflationary era, where it was found that the collapse must have driven the highly squeezed quantum state of the modes of the inflaton fields towards states with relatively well defined values of either the field operator or the canonically conjugated momentum operator [44]. We still do not know examples of generic operators, which reduce to the smeared position operators in one regime, and to the above operators in the other. This question will be the focus of future research.

Moreover, we have shown that the non-conservation of energy required to get an effective cosmological constant of the magnitude of the one observed today would not imply any visible effect on the fluid equation of state. Physically, the energy created, or lost, would produce effects that accumulate in $\Lambda^{\rm eff}$ whereas its backreaction on ordinary matter or radiation decreases due to expansion of the universe. In that framework, one could see the cosmological constant today as a record of the energy-momentum non-conservation in the history of our universe.

It is worth noting that although, the quantitative results in this paper have been obtained in the context of a particular model of modified quantum mechanics where violations of energy-momentum conservation arise, the broad results of our analysis are more general. On the one hand, other models of spontaneous collapse will lead to qualitatively similar results (as they depend only on the general property of (9) of not conserving the energy). On the other hand, this kind of analysis could also be applied to phenomenological models where violations of energy-momentum conservation are incorporated from the onset by specific choice of the field action describing the coupling of unimodular gravity with matter.

Finally, from the point of view of wave-function collapse models, one can take advantage of the huge cosmological time to test violation of energy conservation, and thus, put constraints on the collapse rate.

We conclude by reiterating that at this point we have no dynamical reduction theory mature enough to allow a reliable calculation applicable to the complete cosmological evolution of our universe, beginning from, say, the start of inflation. We look forward to progress in this line of research which we have now shown, might offer a path to predict, using independent physical ingredients, the value of the dark energy.

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